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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01
Paper 1

General comments

The overall performance of candidates on this Paper was both pleasing and encouraging. There were few very poor scripts and the Paper enabled candidates to demonstrate accurately what they had been taught. The standard of numerical and algebraic manipulation was good and in many cases impressive. **Question 6** was the only question on the Paper that caused candidates real problems and candidates need to be aware of the implication of the word "exact".

Comments on specific questions

Question 1

Most candidates coped confidently with the binomial theorem, though many preferred to show the whole expansion prior to selecting the term that is independent of *x*. Most did select the correct term, but others, by giving the answer as the whole expansion, did not appreciate the term "independent". The error of expressing $\left(x + \dfrac{3}{x}\right)^4$ as $x\left(1 + \dfrac{3}{x^2}\right)^4$ was common.

Answer: 54.

Question 2

This was well answered with most candidates scoring highly. The solution of the two simultaneous equations in *a* and *r* was accurate, though $r^2 = \dfrac{18}{8}$ instead of $\dfrac{8}{18}$ was a common error. Loss of an accuracy mark through taking *r* as 0.66, 0.67 or even 0.7 was common. Evaluation of the sum to infinity was accurate and only very occasionally was an arithmetic progression used instead of a geometric progression.

Answers: **(i)** *a* = 27, *r* = $\dfrac{2}{3}$; **(ii)** 81.

Question 3

(i) Many candidates failed to use trigonometry to express *QR* as $r\tan\theta$.

(ii) This was accurately done with candidates using trigonometry to evaluate *QR* and *OR* and most realised that $PR = OR - r$. A common error was to assume that the perimeter of *QPR* was the same as the difference between the perimeters of triangle *OQR* and sector *OQP*. Misuse of radians was still common in many cases.

Answers: **(i)** Proof; **(ii)** 34.0 cm.

Question 4

This proved difficult with many candidates failing to realise the need to integrate. Many attempts were seen in which the equation of the curve was taken to be the same as the equation of the tangent, with $m = \dfrac{dy}{dx} = 3$ being used in $y = mx + c$. Attempts at integration were variable with many candidates failing to divide by the differential of (1 + 2x). Many others failed to include the constant of integration.

Answers: **(i)** $y = \dfrac{1}{3}(1 + 2x)^{\frac{3}{2}} + 2$; **(ii)** $\dfrac{7}{3}$.

Question 5

This was very well answered. Virtually all candidates correctly used the equations connecting sine, cosine and tangent to obtain the required result. The solution of this equation was generally correct, though $\sin\theta = -\dfrac{1}{2}$ and 2 was a common error. Many candidates obtained full marks.

Answer: **(i)** Proof; **(ii)** 30°, 150°.

Question 6

This was poorly answered with many candidates failing to cope with, or to recognise, the trigonometry needed for the two parts. Failure to express *AC* and *BC* in terms of *l* presented problems and even when candidates used Pythagoras to find *AB*, use of decimals for cos 30°, instead of $\dfrac{1}{2}\sqrt{3}$, meant that accuracy marks were lost. Candidates must realise that asking for an exact answer precludes the use of decimals in this case. In part **(ii)** only about a half of all candidates realised that use of tangent in triangle *ABC* led directly to the answer. Again use of decimals, particularly for cos30°, meant that the final answer was not obtained.

Answers: **(i)** $AC = \dfrac{1}{2}l\sqrt{3}$, $BC = l$, Proof; **(ii)** Proof.

Question 7

This was well answered with candidates showing a pleasing understanding of the use of scalar product. In both parts, evaluation of the scalar product was accurate. Use of $|a| = \sqrt{(2^2 - 2^2 + 1^2)}$ was seen and the final answer to part **(i)** was often expressed to the nearest degree, rather than to one decimal place, or inaccurately as 103.7°. The main error however was the loss of this accuracy mark through thinking that the direction between two vectors was always an acute answer. Part **(ii)** was also well answered, though a small proportion of attempts were seen in which the scalar product was equated to ±1 rather than to 0.

Answers: **(i)** 103.8°; **(ii)** $-\dfrac{3}{11}$.

Question 8

Parts **(i)** and **(ii)** presented little difficulty apart from the occasional error of $\dfrac{d}{dx}(x^3) = 2x^2$ and careless slips in the solution of the quadratic equation resulting from $\dfrac{dy}{dx} = 0$.

Part **(iii)** presented more problems with many candidates failing to appreciate that $y = 0$ on the *x*-axis.

Answers: **(i)** $3x^2 + 6x - 9$; **(ii)** $x = -3$ or 1; **(iii)** $k = -27$ or 5.

Question 9

This was generally well answered with many completely correct solutions. Simple errors in evaluating the gradient of AB were common, but most candidates accurately used the formulae $m_1m_2 = -1$ and $y - k = m(x - h)$.

Method marks in part **(ii)** and **(iii)** were nearly always gained, though "$d^2 = (x_2 + x_1)^2 + (y_2 + y_1)^2$" or "$d^2 = (x_2^2 - x_1^2) + (y_2^2 - y_1^2)$" were both seen. Several candidates wasted considerable time in part **(i)** by finding the equation of AB and in part **(iii)** by finding the coordinates of D. The final answer was accepted in either decimal or surd form, though $2\sqrt{20} + 2\sqrt{180}$ needed further simplification.

Answers: **(i)** $2y = x+11$; **(ii)** $C(13, 12)$; **(iii)** 35.8 or $16\sqrt{5}$.

Question 10

(i)   This part caused problems with many candidates failing to gain any credit through failing to appreciate the need to use calculus. A surprising number of attempts were seen in which the gradient of $y = 2\sqrt{x}$ was taken to be 2. A depressing number of candidates took the gradient of the normal to be $-\sqrt{x}$ and the equation of the normal to be $y - 4 = -\sqrt{x}(x - 4)$.

(ii)   This part was very well answered with most candidates realising the need to integrate. The standard of integration and the subsequent use of limits were good but a significant number used the limits 2 to 4 rather than 1 to 4.

Answers: **(i)** $y + 2x = 12$; **(ii)** $9\frac{1}{3}$.

Question 11

The candidate's responses to this question, particularly to part **(v)**, were much better than in recent examinations.

(i)   Most of the attempts were correct, though failure to take the "2" out of the expression caused problems for weaker candidates.

(ii)   This part was well answered with most candidates realising that the least value of y was c and that this occurred at $x = -b$. Others preferred to use calculus and were generally correct.

(iii)   This part presented more problems though most candidates realised that the limit values of the range occurred at $x = 2$ and $x = -6$. Too often, however, the required set of x values was given as "$-6 \leqslant x \leqslant 2$" or as "$x \geqslant 2, x \geqslant -6$".

(iv)   Very few candidates appreciated that the inverse of a function only exists if the function is one-one and that the smallest value of k corresponds to the value of x at the turning point.

(v)   This part was well answered with at least a half of all candidates realising the need to use the answer to part **(i)** to express x in terms of y and then to interchange x and y. Writing $\sqrt{\dfrac{x + 18}{2}}$ as $\dfrac{\sqrt{x + 18}}{2}$ was a common error.

Answers: **(i)** $a = 2$, $b = 2$, $c = -18$; **(ii)** $x = -2$, $y = -18$; **(iii)** $x \geqslant 2$, $x \leqslant -6$; **(iv)** -2; **(v)** $f^{-1}(x) = \sqrt{\dfrac{x + 18}{2}} - 2$.

General comments

Very few candidates scored highly on this Paper and a significant minority were unable to score many marks. In part, this was due to a failure to use the many helpful formulae and results given in list MF9.

Candidates invariably attempted all questions (except **Question 4**) and there was no evidence of time being insufficient to complete the Paper. **Question 2**, **Question 5 (i)** and **(ii)**, and **Question 7 (i)** were generally well attempted, but there were few good responses to **Question 3**, **Question 4**, **Question 5 (iii)**, **Question 6** and **Question 7 (ii)**. Many candidates displayed little knowledge of the background to these latter questions, and key sections of the syllabus such as basic differentiation and integration were beyond the ability of a sizeable minority. **Question 4**, most particularly, often produced responses that earned no more than one or two marks.

The Examiners were pleased by the clear, well presented, nature of the candidates' work, and the questions were usually attempted sequentially, though **Question 4** was often left until the end as most candidates were obviously troubled by it.

Comments on specific questions

Question 1

Candidates attempted this question by one of two methods. Those who squared each side and solved the resulting quadratic equation, or inequality, usually scored at least 3 of the 4 marks, but were often unable to choose correctly which intervals on the x-axis were relevant; use of a single value, say $x = 0$, in the original inequality is an excellent guide as to which interval(s) are the correct one(s). Other candidates attempted to remove the original inequality/modulus signs but only rarely obtained more than a single mark, by showing that $x = -1$ was a crucial point. The Examiners would advise candidates to use the more structured approach of squaring each side.

Answers: $x < -1$, $x > \dfrac{1}{5}$.

Question 2

This was a popular question and was well attempted. Errors arose when values $x = +1$, $+2$ instead of the correct $x = -1$, -2 were substituted into the cubic polynomial and also when the remainder on division by $(x + 2)$ was accidentally set equal to zero, instead of the given remainder, -5. Several candidates also misread $2x^3$, as $2x^2$, and/or ax^2 as ax. Long division was rarely used, albeit usually successfully.

Answers: $a = 3$, $b = -1$.

Question 3

(i) Very few candidates found $9^x = y^2$; more often, $9^x = 2y$ or $9^x = 3y$ were seen. An erroneous answer to part **(i)** unfortunately makes it impossible to proceed to score in part **(ii)**, as no viable problem can then be attempted, as a follow-on from $9^x = 2y$ or $3y$.

(ii) Candidates obtaining a correct answer to part **(i)**, and many who began afresh without reference to part **(i)**, usually scored full marks. Any wrong answer to part **(i)**, if used in part **(ii)**, produced intractable problems, which leads the majority to appeal to the false results $\ln(a + b) = \ln a + \ln b$ and/or $\ln\left(\dfrac{a}{b}\right) = \dfrac{\ln a}{\ln b}$, in context.

Answers: **(i)** y^2; **(ii)** $x = -1$, $-\dfrac{\ln 2}{\ln 3}$.

Question 4

(i)   Only a handful of correct pairs of graphs were seen. Although the sketch of $y = \sin x$, $0 < x < \frac{\pi}{2}$, was familiar to almost all candidates, very few even attempted to sketch $y = x^{-2}$; those who attempted a sketch rarely produced a graph of the correct basic shape.

(ii)   Having evaluated $y = \sin x$ and $y = x^{-2}$ at $x = 1, 1.5$ many solutions made no comparison of their values. Other solutions stated that the function $f(x) = \sin x - x^{-2}$ changes sign between $x = 1$ and $x = 1.5$, but no numerical evidence was produced; this behaviour pattern is indeed suggested by the question.

(iii) When attempted, this was well done.

(iv) As on previous occasions when such an iteration has been set, most candidates treated the angle x as being measured in degrees. The Examiners stress once again that, as in all rules for differentiation and integration of trigonometric functions, everything is based on the premise that angles are measured in **radians**, unless degrees are explicitly indicated. Those who did not make this mistake tended to score full marks for this part.

Answer: **(iv)** 1.07.

Question 5

(i)   Candidates either correctly expanded on both sides, with occasional sign errors between terms, or incorrectly states that $\cos(x - 30°) = \cos x - \cos 30°$, etc. Although the given result used surds, many candidates set $\cos 30° = 0.866$ instead of $\frac{1}{2}\sqrt{3}$.

(ii)   Examiners were surprised how often the result from part **(i)** was wrongly simplified to $\tan x = \frac{\sqrt{3}}{2}, \frac{2}{\sqrt{3}}$ or $\frac{1}{2\sqrt{3}}$. Those who correctly set $\tan x = 2\sqrt{3}$ proceeded successfully to find the two appropriate solutions.

(iii) Candidates seemed very puzzled by this. It was common to see $(2\cos^2 x - 1)$ replacing $\cos 2x$, and then $2\cos^2 x = 1$, etc. Others used the basic angle of $73.9°$ from part **(ii)** and calculated $\cos(2 \times 73.9°)$, even though an exact answer was requested. It was expected that candidates would use the result from part **(i)** to obtain an exact value for $\cos x$ or $\sin x$ and use this to evaluate $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$.

Answers: **(ii)** 73.9°, 253.9°; **(iii)** $-\frac{11}{13}$.

Question 6

(a)   Few candidates could correctly integrate $\sin 2x$; a sizeable minority did not know the value of $\int \cos x \, dx$, with functions such as $-\sin x$, $\frac{\cos(x^2)}{2}$, $\frac{\cos x}{x}$ often quoted. Use of list MF9 would have avoided these errors. Further the values of $\cos 2x$ and $\sin x$ at $x = 0$, $\frac{\pi}{2}$ were often wrongly given as decimals and not integers.

(b)(i) Almost all solutions involved $\int_1^p \frac{dx}{x+1}$, but few candidates could integrate $(x + 1)^{-1}$, which was often rewritten as $(x^{-1} + 1)$. Functions such as $\frac{1}{\frac{1}{2}x^2 + x}$ or $\ln x + 1$ were common. The correct function $\ln(x + 1)$ was frequently rewritten as $\ln x + \ln 1$. The final form, $\ln(p + 1) - \ln 2$, was popularly rewritten as $\ln p + \ln 1 - \ln 2$ or $\frac{\ln(p+1)}{\ln 2}$.

(ii) Only those candidates with a logarithmic solution to part **(i)** could successfully score in part **(ii)**.

Answers: **(a)** $\left[-\dfrac{1}{2}\cos 2x + \sin x \right]_{0}^{\frac{\pi}{2}} = 2$; **(b)(i)** $\ln(p + 1) - \ln 2$, **(ii)** 13.8.

Question 7

(i) This part was often well done, with at least a correct derivative of $3y^2$ or $-2xy$. Weaker candidates tried vainly to express y explicitly in terms of x; no actual differentiation followed.

(ii) A high proportion of solutions involved setting $y' = 0$, but many believed that this implied that $y - 2x = 3y - x$ or that $3y - x = 0$. Other candidates believed that $y' = 1$ or -1 if the tangent is parallel to the x-axis.

The main problem, however, was that the majority of candidates believed not only that $y - 3x = 0$, or $3y - x = 0$, or $y - 3x = 3y - x$, but that x (or y) was also zero. It was difficult to see where such a false premise arose, but it permeated most solutions. Only a few candidates set $y = 2x$ into the original equation of the curve and then solved the resulting quadratic equation in x (or y).

Answers: **(ii)** $(1, 2)$ and $(-1, -2)$.

| **Paper 9709/03** |
| **Paper 3** |

General comments

The standard of work by candidates on this Paper varied widely and there was a corresponding range of marks from zero to full marks. All the questions appeared to be accessible to candidates who were fully prepared and no question seemed to be of unusual difficulty. Moreover, adequately prepared candidates appeared to have sufficient time to attempt all the questions. Overall, the least well answered questions were **Question 3**(logarithms), **Question 4**(exponential function) and **Question 7**(iteration). On the other hand **Question 2**(integration by parts), **Question 6**(partial fractions) and **Question 10** parts **(i)** and **(ii)**(vector geometry) were felt to have been done well.

The detailed comments that follow inevitably refer to mistakes and can lead to a cumulative impression of poor work on a difficult Paper. In fact there were many scripts showing very good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on specific questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the only 'correct' answer.

Comments on specific questions

Question 1

Most candidates found at least one of the correct critical values, 4 and 5, in the course of their work. However errors in handling inequalities prevented a substantial number from completing the question successfully. Failure to reverse an inequality when dividing by a negative quantity was a common error.

Answer: $4 < x < 5$.

Question 2

This was often well answered, though some candidates were unaware of the correct meaning in this context of the adjective 'exact'. Examiners noted that some candidates started correctly but failed to deal with the second integral properly, by first reducing the integrand to $\frac{1}{2}x$ before integrating.

Answer: $2\ln 2 - \frac{3}{4}$.

Question 3

The first part was poorly done. Common errors included the incorrect assumption that $\log(x + 5) = \log x + \log 5$, and failure to realise that $2 = \log_{10} 100$. In the second part, the stronger candidates observed that x needed to be positive but others often presented both roots of the quadratic equation as possible solutions to the problem.

Answers: **(i)** $x^2 + 5x - 100 = 0$; **(ii)** 7.81 .

Question 4

Many attempts which started with a correct first derivative foundered because of an inability to solve the indicial equation that followed. Candidates were often unable to handle the term in e^{-2x} correctly and here, as in **Question 3**, there was much unsound work with logarithms. However most had an appropriate method for determining the nature of a stationary point, the majority using the second derivative.

Answers: **(i)** $\ln 2$; **(ii)** Minimum point.

Question 5

The first two parts were generally well done.

(i) In this part the value of R was almost always correct, but incorrect values of α arising from $\tan \alpha = \frac{4}{3}$ or $\tan \alpha = -\frac{3}{4}$ were given from time to time. Examiners also found that α was not always given to the accuracy requested in the question.

(ii) The smaller root was usually obtained correctly, but some candidates lacked a sound method for the larger root and commonly gave $119.6°$.

(iii) There was a widespread failure to answer this part correctly. Answers such as $\frac{1}{11}$ and $\frac{1}{7}$ were more common than the correct one.

Answers: **(i)** $5\sin(\theta - 36.87°)$; **(ii)** $60.4°$ and $193.3°$; **(iii)** 1.

Question 6

Candidates seemed generally well prepared for this question and attempted it well. However, numerical errors in finding the numerators of the partial fractions were quite common and only the ablest candidates traced them back when they failed to obtain the given answer to part **(ii)**.

(i) It was rarely evident from the scripts that candidates were checking their answers to this part either by recombining the fractions to form f(x) or by substituting a value for x, but in questions of this type this seems a worthwhile precaution.

(ii) The most common source of error was in the expansion of $(2 - x)^{-1}$. A minority worked *ab initio* with f(x), either expanding $(6 + 7x)(2 - x)^{-1}(1 + x^2)^{-1}$ or, occasionally, carrying out a long division.

Answer: **(i)** $\dfrac{4}{2 - x} + \dfrac{4x + 1}{1 + x^2}$.

Question 7

(i) Attempts at this part varied considerably in length and success. Candidates who used the cosine or sine rule in triangle OAB were much less successful than those who observed that $\sin\alpha = \dfrac{99}{2r}$.

(ii) Examiners reported that many candidates failed to produce sufficient accurately calculated evidence to justify the given statement about the root.

(iii) The work in this part was even more disappointing. Few candidates seemed to know that the solution involved replacing the iterative formula with an equation and showing that this equation was equivalent to that given in part **(i)**.

(iv) However, this part was frequently correctly done. The most common error here was to carry out the calculations with the calculator in degree mode rather than in radian mode.

Answer: **(iv)** 0.245 .

Question 8

(a) Though some candidates omitted this part, most were familiar with a method for finding the square roots of a complex number and nearly all chose to work with a pair of simultaneous equations in x and y. Some made algebraic errors when eliminating an unknown or in solving their quadratic in x^2 or y^2, but most showed an understanding of the method.

(b) In part **(i)** almost all attempted to multiply the numerator and denominator by $2 - i$ but errors in simplification were common.

Part **(ii)** was generally done well. There were a variety of acceptable answers to part **(iii)** and few candidates were able to find one.

Answers: **(a)** 1 + 2i and $-1 - 2$i; **(b)(i)** $\dfrac{1}{5} + \dfrac{7}{5}$i , **(iii)** $OC = \dfrac{OA}{OB}$.

Question 9

(i) There were many correct solutions in this part. A minority of candidates merely showed that the given differential equation was satisfied when $a = 5$. This does not show that a satisfies the equation at all times.

(ii) Although most attempts at the partial fractions were successful, a substantial number of candidates failed to relate their partial fractions to the differential equation. Omission of the constant of integration was a frequent error and candidates often failed to complete this section by obtaining t in terms of a. Here, as in **Question 3** and **Question 4**, there were instances of unsound work with logarithms.

(iii) Most candidates correctly let $a = 9$ but the unrealistic substitution $a = 0.9$ was also seen quite frequently.

Answers: **(ii)** $t = 25\ln\left(\dfrac{a}{10-a}\right)$; **(iii)** 54.9 days.

Question 10

(i) This part was very well answered.

(ii) In this part the most popular method was to write down parametric equations of the two lines and examine simultaneous equations obtained by equating the corresponding x, y, and z components. Having used two equations to calculate one of the parameters, many candidates went on to calculate the other and check that all three equations were satisfied simultaneously, but some failed to carry out this essential final step. A fairly common error was to use the same parameter in both the vector equations.

(iii) Only the strongest candidates devised a valid method for this part This usually began by finding the parameter of the point *N* on the line *AB* such that *PN* was perpendicular to *AB*, though here, as in part **(ii)**, Examiners encountered a pleasing variety of approaches including, for example, the use of the orthogonal projection of *AP* (or *BP*) onto *AB*.

In general, Examiners felt the standard of work on this topic was encouraging. It could be improved if candidates persistently checked for arithmetic errors (especially sign errors) and, when searching for a method, as many were in part **(iii)**, they drew a simple diagram to illustrate the problem.

Answer: **(i)** 45.6°.

> **Paper 9709/04**
>
> **Paper 4**

General comments

Many candidates made good attempts at some of the questions, but relatively few candidates answered well across the full range of topics represented by the questions in this Paper. Some candidates were clearly not prepared for this Paper and scored very low marks.

Some candidates failed to attain the accuracy required by the rubric because of premature approximation. Frequently occurring cases included 8/**(0.77 – 0.34)** = 18.6 and 8/**0.42** = 19.0 in **Question 3 (i)** with consequent errors and further premature approximation of trigonometrical values in part **(ii)**, and $0.25 \times 12 \times$ **0.91** = 2.73 in **Question 6 (i)(a)**, usually followed by 1.95 in part **(b)** and 3.95 in part **(c)**.

Some candidates gave answers to insufficient accuracy, the most common of which were 19 and 30 in **Question 3 (i)** and **(ii)** respectively, 0.47 or 0.5 in **Question 5 (ii)**, and 2.7, 2 and 4 in **Question 6 (i)(a)**, **(b)** and **(c)** respectively.

An apparent lack of understanding of the concept of displacement among many candidates is evident from the high frequency of the answer of 200 m in **Question 2 (ii)** and from attempts at solving $s(t) = 2 \times 800$ in **Question 7 (iv)**.

Comments on specific questions

Question 1

This was found to be a straightforward starter question with most candidates scoring all three marks. The most common mistake was to omit, or to assign the wrong sign to, the resistance to motion in applying Newton's second law.

Answer: 0.2 ms^{-2}.

Question 2

Part **(i)** of this question was well attempted with most candidates scoring both marks. However some candidates, having calculated the approximate distance as 154 m, gave the reason for it being an underestimate that the man was already running at time *t* = 0.

In part **(ii)** very many candidates calculated the total distance run by the man (200 m), instead of his distance from *A*. Some candidates gave the answer as 160 m, believing that the man was moving towards *A* for 30 < *t* < 35 and away from *A* for 35 < *t* < 40.

Answer: **(ii)** 120 m.

Question 3

Many candidates demonstrated by their answers considerable confusion as to how to proceed in this question.

Some candidates tried to use Lami's theorem, without attempting to reduce the problem to one of three forces by, for example, combining the tensions in the two parts of the string to give a resultant of $2T\cos35°$ acting at 15° above the horizontal.

Some candidates applied Lami's theorem after changing the question by rotating the given diagram clockwise through 90°, so that A and B are at the same horizontal level and X acts vertically downwards. None pointed out that the configuration is now impossible given that R is smooth. It may be that the motivation for such candidates was the recognition of the need to reduce the problem to one of three forces (T, T and $X + 8$) in order to apply Lami's theorem.

Almost all candidates who made scoring attempts resolved forces on R vertically and horizontally, or at least one of these. Errors arising in doing this were to include a spurious 'normal reaction' acting vertically upward on R, taking the tensions in the two parts of the string to be different, and writing $F_X = X - T\cos50° - T\cos20°$ and $F_Y = T\sin50° - T\sin20° - 0.8g$ without ever setting $F_X = 0$ or $F_Y = 0$.

Many candidates scored only 5 out of 6 marks for a basically correct solution because inaccuracies arose from using insufficiently accurate values of the trigonometrical ratios.

Answers: **(i)** 18.9 N; **(ii)** 29.9.

Question 4

Part **(i)** of this question was well attempted, although many candidates found the difference in the maximum heights of the particles.

Part **(ii)** was also well attempted, although many candidates calculated the height of A at the instant when B is 0.9 m above the ground.

A surprisingly significant proportion of candidates used $a = +g$ where $a = -g$ is appropriate.

Answers: **(i)** 1.5 m, **(ii)** 1.05 m.

Question 5

Part **(i)** of this question was almost always answered correctly.

In part **(ii)** almost all candidates resolved forces along the plane for both of the cases illustrated in Fig.2. However very many mistakes were made, the most common of which were:

F taken to be in the same direction in both cases, leading trivially to $X = 0$,

F taken to be in the wrong direction in both cases, leading to a negative coefficient of friction,

the omission of F in one or both of the cases,

the inclusion of a term ma in both cases,

writing $F_1 = 150\sin35° - X - F$ and $F_2 = 150\sin35° - 5X + F$ without ever setting $F_1 = 0$ and $F_2 = 0$.

Answer: **(ii)** $X = 28.7$, coefficient of friction = 0.467.

Question 6

Although the wording of the relevant part of the syllabus is 'understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component', it is clear that very many candidates did not know what was required of them in part **(i)(a)** of this question. Frequently the answer was given as $R = 10.9$ or as $C = 11.2$.

This shortcoming was not a barrier to answering parts **(b)** and **(c)** correctly, as many candidates did. The most common error in part **(b)** was to omit either the weight component or the frictional component in applying Newton's second law, and in part **(c)** the most common error was the omission of the work done against friction by candidates who considered energy.

Part **(ii)(a)** was almost always answered correctly.

Very few candidates scored all three marks in part **(ii)(b)**. The common mistakes were to apply a formula for motion in a straight line, and to omit the velocity at the bottom of the slope in applying the principle of conservation of energy.

Answers: **(i)(a)** 2.72 N, **(b)** 1.96ms^{-2}, **(c)** 3.96 ms^{-1}; **(ii)(a)** 36 J, **(b)** 8.70 ms^{-1}.

Question 7

Almost all candidates successfully verified that P comes to rest when $t = 0$ in part **(i)** of this question. Most differentiated successfully to find $a(t)$, but many then found $a(0)$ instead of $a(200)$, presumably because they misunderstood 'starts to return towards' as 'starts from'.

Part **(ii)** was not well done and often omitted. Many candidates' solutions involved integrating the expression for $v(t)$, often followed by the use of a constant acceleration formula to give, for example, $v + 0 = \dfrac{2 \times 800}{200}$ and $v^2 = 0^2 + 2(-0.12)800$. Candidates who correctly obtained 100 as the value of t for which v is a maximum often omitted the calculation of $v_{\max}$.

Part **(iii)** was very well attempted and many candidates obtained the correct answer.

Part **(iv)** was rarely answered correctly. The main errors were to solve $v(t) = 0$ or $s(t) = 2 \times 800$ instead of $s(t) = 0$, to obtain $2 \times 800 = \dfrac{1}{2}(0.12)t^2$ by using a constant acceleration formula, or simply to double the 200 s which P takes while travelling outward from O before starting its return.

Answers: **(i)** 0.12 ms^{-2}; **(ii)** 6 ms^{-1}; **(iii)** 800 m; **(iv)** 300.

| **Paper 9709/05** |
| **Paper 5** |

General comments

Candidates who had a good grounding of the mechanical ideas needed in all parts of the syllabus found that all questions were accessible. Even moderate candidates found that they could make very good progress into some of the questions.

There were very few candidates who had difficulty in finishing the Paper through lack of time.

Many candidates seemed to be uncertain about the forces acting on a body. The misunderstandings included omission as in **Question 2 (b)**, the wrong direction as in **Question 4 (c)**, the effect of the forces acting on a body as in **Questions 4**, **5** and **6**. These points will be dealt with later in this report.

Comments on specific questions

Question 1

There was a surprising lack of overall success with this question. The angular speed was frequently multiplied by 6, 8 or more often than not, 12. The latter figure came from either $\frac{1}{2}AC$ or the length of the median through *B*. It would seem that not only was the idea of the centre of mass of a triangular lamina not understood, but also that there was a lot of misreading of the question in having the lamina rotating about *A*. There were a lot of involved calculations to find the length of the median through *B*, and only rarely was the length found with the statement 12tan45°. Again, instead of a straightforward substitution into the formula *v* = *rω*, many equated the two acceleration formulae for circular motion to find *v*.

Answer: 20 cm s^{-1}.

Question 2

Those who realised that the key to this question involved taking moments about *B* usually got the maximum five marks. Candidates should have realised that, for the rod to be in equilibrium, there must be a force acting on it at *B*, even though it may have played no part in the solution. Had they done so, many solutions would not have had as their starting point the incorrect statement *T*sin30° = 10*g* in an alleged attempt to resolve vertically. Naturally taking moments about *B* meant that the force at *B* did not appear in the solution.

In part **(ii)** nearly all candidates knew that they had to apply Hooke's Law to obtain an equation in either the extension only or the natural length only, so that only the poorest candidates failed to score at least two marks in part **(ii)**.

Answers: **(i)** 100N; **(ii)** 2 m.

Question 3

Regrettably the majority of the candidates failed to read the question properly in that *y* was defined as the vertically *upward* displacement. Hence the equation of the trajectory invariably had the negative sign missing. Many candidates correctly derived the general trajectory equation as given in list MF9, but then failed to equate θ to zero.

Part **(ii)** was well answered with the correct angle being obtained with a variety of correct methods. Weak candidates incorrectly divided the 45 m by the horizontal displacement at sea level in an attempt to find the angle. Another longer approach was to consider the flight path in reverse by substituting *x* = 30 m and *y* = 45 m into the general trajectory equation and then solving the quadratic equation in tan*θ*. Most solutions were wrong because candidates substituted the value *v* = 10 m s^{-1}. Had they used the value of the speed at sea level ($\sqrt{1000}$ ms^{-1}) the correct answer would have been obtained.

Answers: **(i)** $y = -\dfrac{x^2}{20}$; **(ii)** 71.6°.

Question 4

There was a welcome improvement in the amount of success achieved with the circular motion problem compared with previous years' A Level attempts, with many all correct solutions. The most frequent error by the moderate candidates was to assume that the tensions in both strings was the same so that an opening statement 2*T*cos30° = 0.5(20) was often seen. Had these candidates bothered to look at the vertical motion, it would have led to a resultant vertical downward force of 5 N, and so it would have been impossible for the ball *B* to rotate in the same horizontal circle. Weaker candidates often ignored the tension in the lower string and, even if it was there, it was often in the wrong direction.

Answer: 10.8 N.

Question 5

Better candidates realised that the key to solving this question was an application of the energy principle and, on the whole, scored well in both parts of the question. However the usual opening gambit of a substantial number of candidates was to assume that, at the lowest point, the resultant force parallel to the slope was zero, leading to the incorrect statement $\dfrac{1.5x}{2}$ = 0.075$g$sin30°. Had this idea been correct, the particle *P* would have remained at rest at its lowest point as there would not have been any resultant force to start moving *P* up the plane.

For those who knew that part **(ii)** depended on an application of Newton's Second Law of Motion, the most frequent error was to ignore the tension in the string when setting up an equation. Those candidates who did not use the correct ideas usually followed up their mistaken ideas in part **(i)** by attempting to find a value for the speed at the lowest point by misusing the energy principle. A non-zero value of the speed was found, despite the fact that they had correctly taken its value to be zero in part **(i)**. They then used the equations for constant acceleration under the mistaken belief that the acceleration would be constant for all of the subsequent upward motion. Apart from the failure to distinguish between an instantaneous acceleration and one which remained constant throughout the motion, there was also the failure to recognise that, in the subsequent motion up the plane, as the tension varied, in accordance with Hooke's Law, so must the resultant force on *P* and hence a constant acceleration would be impossible.

Answers: **(i)** 4 m; **(ii)** 15 m s^{-2}.

Question 6

Candidates who were familiar with the calculus approach to this type of problem often scored very well in parts **(i)** and **(ii)**. Regrettably many of the rest sought a solution dependent on the use of the constant acceleration equations. Perhaps with all questions involving the movement of a particle, the first question that a candidate should ask is "Is the resultant force acting on the body constant?". If the answer is "No" then, since a varying force produces a varying acceleration, the constant acceleration equations cannot be used. Here, as the retarding force depends on the velocity, this force cannot be constant and must therefore lead to a varying acceleration.

In part **(ii)** the expected approach was merely to replace *v* in part **(i)** by $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ and integrate again. Many made more work for themselves by starting again with the acceleration equal to $\dfrac{\mathrm{d}v}{\mathrm{d}t}$. Having got *v* as a function of *t*, this equation was then integrated again to produce the required result.

Part **(iii)** was not well answered. It was not sufficient merely to state that when *t* is infinite, *x* = 100. The possibility existed for *x* to exceed 100 at some finite time and then converge to 100 from above. All that was required was a recognition that, as $\exp\left(-\dfrac{t}{20}\right)$ is positive for all values of *t*, (1 − $\exp\left(-\dfrac{t}{20}\right)$) is less than unity and hence *x* is less than 100.

Answers: **(i)** $v = 5 - \dfrac{1}{20}x$; **(ii)** $x = 100\,(1 - \exp\left(-\dfrac{t}{20}\right))$ (or equivalents).

Question 7

There were many all correct solutions for *x* and *y* in part **(i)**. More often than not, those who made errors in either the areas of the rectangles or the distances of their centre of masses from *OX* or *OY* had only themselves to blame because of their poor sketches in which the sides of the rectangles were not clearly defined.

Parts **(ii)** and **(iii)** were not very well answered, usually due to a lack of clear explanation as to how the inequalities, as opposed to the equalities, were arrived at. Although it was generally realised that the weight acted through *O* in part **(iii)**, the diagrams for part **(ii)** tended to show that candidates were unaware that the critical toppling position had to be considered in the case of sliding before toppling too. Although it did not enter into the calculation, most sketches for part **(ii)** indicated that few seemed to be aware that, on the point

of toppling, the normal component of the force of the plane on the lamina acted at *O*. Perhaps those candidates who obtained $\tan\left(\dfrac{x}{y}\right)$ for no toppling by considering moments about *O* were lucky in that the non-considered normal component passed through *O* despite the fact that their sketches often showed the point of application of this force to be somewhere between *O* and *X*. A common failing in this question, or indeed any questions involving frictional forces, is that candidates quote $F < \mu R$ without any added qualification. It would be equally true to write $F \geqslant \mu R$ provided that there is the added qualification "the body slides, or is about to slide".

Answers: **(i)** *x* = 8.75, *y* = 6.25; **(iii)** $\dfrac{5}{7}$.

Paper 9709/06

Paper 6

General Comments

This Paper produced a wide range of marks. Many Centres, however, entered candidates who had clearly not covered the syllabus and thus a large number failed to reach the required standard. Premature approximation leading to a loss of marks only occurred in a few scripts, most candidates realising the necessity of working with, say, $\sqrt{22.5}$ instead of 4.74.

Candidates seemed to have sufficient time to answer all the questions, and only the weaker candidates answered questions out of order.

Comments on specific questions

Question 1

This question was well answered with generally full marks. It was good to see almost every candidate solving two simultaneous equations correctly.

Answers: **(ii)** *a* = 0.15, *b* = 0.3.

Question 2

Some candidates drew a possibility space for the sum of **two** dice scores and could not adapt it to 3. However, the majority found at least some of the options if not all of them.

Answer: **(ii)** $\dfrac{5}{72}$.

Question 3

This was well done with pleasing knowledge of the normal distribution. A few candidates lost marks by premature approximation. In part **(ii)** many candidates looked up 0.9 backwards in the tables and did not appear to use the critical *z*-values given at the foot of the page. They then went on to use the wrong sign. Candidates who looked up 0.9 or 0.1 forwards in the table gained no marks for part **(ii)**.

Answers: **(i)** 0.334; **(ii)** 49.9.

Question 4

This was the weakest topic for most candidates. There were numerous interesting ways of obtaining the answer given (28) to part **(i)**. Many candidates failed to see any connection between parts **(i)** and **(ii)**, and only the very best candidates were happy with part **(iii)**, which perhaps gave an answer too large for their minds to accept.

Answers: **(ii)** 162; **(iii)** 688 747 536 or 689 000 000.

Question 5

Rather surprisingly, part **(ii)** was better answered than part **(i)**. Some answers to part **(i)** involved 3 games, not 2, but most candidates recognised this as conditional probability and produced the appropriate formula.

Answers: **(i)** 0.429; **(ii)** 0.31.

Question 6

Parts **(i)** and **(ii)** proved difficult for many candidates. They had trouble appreciating that a box of 10 with equal numbers of chocolate and cream biscuits meant that there were 5 of each. Some found the probability of 5 of each correctly, but then doubled or squared it, thus losing a mark. Part **(iii)** however was well done and most candidates managed the normal approximation with continuity correction and gained credit for this even though the initial probability may have not been quite correct, although many candidates who could not attempt part **(i)** or part **(ii)** produced the correct probability of 0.25 for this part.

Answers: **(i)** 0.0584; **(ii)** 0.307; **(iii)** 0.829.

Question 7

This was well done with many Centres having candidates producing good box-and-whisker plots. Alas some candidates did not label their axes, did not read that both plots had to be on a single diagram, chose non-linear scales, did not use a ruler and generally lost 4 straightforward marks. The comments given were better than usual with many candidates making sensible comments on skewness, spread, or average weights. A few digressed to 'healty diets' or '3rd World Countries' rather than sticking to the statistics of weights. Different ways of finding the median were taken into account, since some textbooks use different ways, and hence more than one answer is given as acceptable.

Answers: **(i)**   LQ 72 or 73 or 71.5, Median 78, UQ 88 or 87.75;
(iii)   'people heavier in *P* than *Q*', or 'weights more spread out in *Q* than *P*', or '*P* is negatively skewed, *Q* is positively skewed or more symmetrical'.

Paper 9709/07

Paper 7

General comments

This was an accessible Paper where candidates were able to attempt most, if not all, of the questions. **Question 6** was particularly well attempted with even the weaker candidates scoring highly. **Question 4**, however, was poorly attempted, and very few of even the better candidates were able to score well. Candidates were able to finish the Paper in the allocated time; there was very little evidence of candidates only partially finishing the last question.

On the whole candidates answers were well presented with all necessary working shown (with the exception of **Question 4**). Most candidates kept to the accuracy required (3 significant figures), though there were occasions when candidates were penalised for premature approximation. Candidates are advised to show all stages in their working out, including pre-rounded figures from calculations.

Comments on specific questions

Question 1

This question was answered quite well by the majority of candidates. The most common error made was to use a wrong z-value (often 2.326 instead of 2.576); other errors included confusion between standard deviation and variance. In particular, as the question stated that the variance was 37.4 minutes², where the squared refers to the units, it was noted by Examiners that some candidates incorrectly used 37.4^2 as the variance, possibly therefore, misinterpreting the notation. Although the question clearly stated that the given estimate of the variance was unbiased, some candidates still, however, incorrectly used 119 in their formula.

Answer: $49.8 < \mu < 52.6$.

Question 2

(i) This part was well attempted with the majority of candidates correctly finding that n was 170.

(ii) Most candidates correctly used the Poisson distribution with mean 3.15, although some candidates incorrectly included P(3) in their sum rather than just P(0) + P(1) + P(2). A few candidates ignored the fact that the question asked for a Poisson approximation to be used and proceeded to use the binomial distribution. This was not what the question required and hence very little credit was given.

Answers: **(i)** $n = 170$; **(ii)** 0.390 .

Question 3

This was a reasonably well-attempted question.

(i) Most candidates were able to set up a standardising equation to solve for n, though sign errors were often seen here. It was also surprising to see that many candidates correctly reached the stage $\sqrt{n} = 12.649$ but then stated $n = 3.557$. A few candidates correctly reached 159.9 but did not then round the value up to the next whole number. Other errors included standard deviation/variance mixes.

(ii) It was pleasing to see most candidates stating their null and alternative hypotheses, and only a minority of candidates were penalised for not clearly showing their comparison between the critical value and the test statistic. It was surprising how many candidates re-calculated this test statistic, thus giving themselves a time penalty. Common errors included use of a two-tail test, an incorrect critical value and contradictions within a final conclusion (for example rejecting H_0 but then incorrectly stating that the mean length remained unchanged).

Answers: **(i)** $n = 160$; **(ii)** Significant growth decrease.

Question 4

This question was not well attempted, even by high-scoring candidates. Candidates frequently did not show all the steps in their method, and whilst the question clearly asked for the critical region to be found, most candidates failed to identify it. Many candidates did not show the necessary comparisons with 0.1 (and quite often comparisons with 1.282 were seen by candidates who had no idea how to answer the question). Very few candidates correctly found the probability of a Type I error, showing an inability to apply their knowledge to the situation in the question. A frequent confusion here was to state that the probability was 0.1, equal to the significance level. As this was a discrete distribution this was not the case.

Answers: **(i)** $X = 0$ or 1, Not enough evidence to say road sign has decreased accidents; **(ii)** 0.0477.

Question 5

(i) This part was well answered, with the majority of candidates correctly using a new mean of 5.6. A few candidates omitted to include P(3) in their calculation, and some weaker candidates wrongly attempted a normal distribution.

(ii)   This was also quite well attempted, though some candidates did not seem to fully understand what they were doing. Inclusion of a continuity correction, for example, was often seen as well as confusion between two possible different methods. The most common error was to use N(2.5, 2.5) rather than N(2.5, 2.5/80), and many standard deviation/variance errors were seen.

Answers: **(i)** 0.809; **(ii)** 0.286 .

Question 6

This was a particularly well-attempted question. The majority of candidates successfully answered parts **(i)** and **(ii)**. Part **(iii)** caused a few problems for some candidates in identifying the limits for the integration, common errors being to use a lower limit of 0 or to integrate from 23.55 to 28. Much additional work was seen in part **(iv)** with a large number of candidates actually calculating the value of the median in order to decide which was greater. Credit was given for this, though the easier method was to compare the probability in part **(iii)** with 0.5.

Answers: **(ii)** 23.6; **(iii)** 0.528; **(iv)** Mean is greater.

Question 7

There were many reasonable attempts at the first part of this question, though these were frequently marred by use of the wrong variance (for example $12^2 \times 0.06^2 + 0.3^2$ was often used instead of $12 \times 0.06^2 + 0.3^2$). Part **(ii)** was less well attempted. Many candidates could not find the correct mean and variance to use, often using answers from part **(i)**. The candidates who did successfully use N(0, 0.0072) often then failed to find $P(D < 0.05)$ as well as $P(D > 0.05)$, thus obtaining a final answer of 0.278 rather than 0.556. This question proved to be a good discriminator.

Answers: **(i)** 0.813; **(ii)** 0.556 .